PMM U.S.S.R., Vol. 46, No. 5, pp. 584-589, 1983.
Printed in Great Britain.

0021-8928/83 $7.50+0.00

UDC 62-50

ON A GROUP PURSUIT PROBLEM*

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The nonstationary case of a problem of pursuit by several controlled objects is examined. Investigations of similar kind, having a direct influence on the obtaining of the present results, were carried out in /1 – 8/. Three schemes are suggested for obtaining sufficient conditions for completing the pursuit in finite time from prescribed initial positions. The schemes in Sects.1 and 3 are generalization and refinement of the corresponding results in /8/, while the scheme in Sect.2 is close in form to the one in /7/.

Given a differential game

$$z_i^{\cdot} = A_i(t) z_i + g_i(t, u_i, v), \quad z_i \in E^{n_i}, \quad u_i \in U_i(t), \quad v \in V(t), \quad t \geqslant t_0 \geqslant 0 \tag{0.1}$$

where E^{n_i} is an n_i-dimensional Euclidean space, $A_i(t)$ are n_i th-order square matrices depending continuously on $t \in [t_0, +\infty)$, $U_i(t)$ and $V(t)$ are continuous many-valued mappings, $U_i(t) \subset U_i \subset E^{n_i}$, $V(t) \subset V \subset E^{n_i}$ for all i and $t \geqslant t_0$, where U_i and V are compacta, the functions $g_i(t, u_i, v)$ are continuous in all the variables; here and below the index i takes the values $1, 2, \ldots, m$. The terminal set $M(t)$ consists of sets $M_i^*(t)$ each of which has the representation $M_i^*(t) = M_i^\circ + M_i(t)$, where M_i° are linear subspaces of E^{n_i}, while $M_i(t)$ are continuous convex-valued mappings such that $M_i(t) \subset L_i$ for each fixed $t \in [t_0, +\infty)$, where L_i is the orthogonal complement to M_i° in space E^{n_i}.

We examine the problem of the trajectory $z(t) = (z_1(t), \ldots, z_m(t))$ of the nonautonomous system (0.1) meeting the set $M(t)$ in finite time from the initial position (t_0, z°), $z(t_0) = z^\circ$. We say that game (0.1) can be completed in time $T = T(t_0, z^\circ)$ from the initial position (t_0, z°) if measurable functions $u_i(t) = u_i(t_0, z_i^\circ, v(t))$, $u_i(t) \in U_i(t)$, $t \in [t_0 T]$ exist such that the solution of the system of equations

$$z_i^\circ = A_i(t) z_i + g_i(t, u_i(t), v(t)), \quad z_i(t_0) = z_i^\circ$$

belongs to the set $M_i^*(t)$ at the instant $t = T$ for at least one value of i for any measurable functions $v(t) \in V(t)$, $t \in [t_0, T]$. Here $u_i(t)$ remembers $v(s)$, $t \geqslant s \geqslant t_0$. Three variants of the solution of the given problem are proposed below.

1. Let π_i be an orthogonal projection operator from E^{n_i} onto the subspace L_i. We introduce the many-valued mappings

$$\Phi_i(t, \tau, u_i, v) = \pi_i \Omega_i(t, \tau) g_i(\tau, u_i, v), \; u_i \in U_i(\tau), \quad v \in V(\tau)$$
$$\Phi_i(t, \tau) = \bigcap_{v \in V(\tau)} \Phi_i(t, \tau, U_i(\tau), v), \quad t \geqslant \tau \geqslant t_0$$

($\Omega_i(t, \tau)$ is the matrizant of the system $z_i^{\cdot} = A_i(t) z_i$ /9/)

Condition 1. The sets $\Phi_i(t, \tau)$ are not empty for all $t \geqslant \tau \geqslant t_0$.

From Condition 1 and the assumptions on the parameters of game (0.1) it follows that the many-valued mappings $\Phi_i(t, \tau)$ are measurable in τ and that sections $\varphi_i(t, \tau) \in \Phi_i(t, \tau)$, $t \geqslant \tau \geqslant t_0$, measurable in τ exist. We fix them and we set

$$\zeta_i(t, t_0, z_i) = \pi_i \Omega_i(t, t_0) z_i + \int_{t_0}^{t} \varphi_i(t, \tau)\, d\tau$$

Let α_i be nonnegative real numbers. We denote

$$\alpha_i(t, \tau, t_0, z_i, v) = \begin{cases} \max\{\alpha_i : \{\pi_i \Omega_i(t, \tau) g_i(\tau, U_i(\tau), v) - \varphi_i(t, \tau)\} \cap \\ \quad \{\alpha_i (M_i(t) - \zeta_i(t, t_0, z_i))\} \neq \varnothing\}, & \zeta_i(t, t_0, z_i) \overline{\in} M_i(t) \\ (t - t_0)^{-1}, & \zeta_i(t, t_0, z_i) \in M_i(t) \end{cases} \tag{1.1}$$

$$\lambda(t, t_0, z) = \min_{v(\cdot)} \max_i \int_{t_0}^{t} \alpha_i(t, \tau, t_0, z_i, v(\tau))\, d\tau$$

*Prikl.Matem.Mekhan.,46,No.5,730-736,1982 where $v(\cdot) = \{v(\tau)\colon v(\tau) \in V(\tau),\ \tau \in [t_0, t],\ v(\tau)$ are measurable$\}$. Let $T(t_0, z) = \inf\{t\colon \lambda(t, t_0, z) = 1\}$.

Theorem 1. Let Condition 1 be fulfilled and let function $\varphi_i(t, \tau) \in \Phi_i(t, \tau),\ t \geqslant \tau \geqslant t_0$, measurable in τ, exist such that $T = T(t_0, z^\circ) < +\infty$, where, in fact, the greatest lower bound is achieved. Then the differential game (0.1) can be completed in time $T - t_0$ from a prescribed initial position (t_0, z°).

Proof. Let $v(\tau) \in V(\tau)$ be an arbitrary measurable function, $\tau \in [t_0, T]$. We denote

$$h(T, t, t_0, z^\circ, v(\cdot)) = 1 - \max_i \int_{t_0}^{t} \alpha_i(T, \tau, t_0, z_i^\circ, v(\tau))\, d\tau$$

Let $\zeta_i(T, t_0, z_i^\circ) \overline{\in} M_i(T)$. Then for $\tau,\ t \geqslant \tau \geqslant t_0$, such that $h(T, t, t_0, z_i^\circ, v(\cdot)) > 0$ we select the controls $u_i(\tau) \in U_i(\tau)$ and the functions $m_i(\tau) \in M_i(T)$ from the equations

$$\pi_i\Omega_i(T, \tau)\, g_i(\tau, u_i(\tau), v(\tau)) - \varphi_i(T, \tau) = \alpha_i(T, \tau, t_0, z_i^\circ, v(\tau))(m_i(\tau) - \zeta_i(T, t_0, z_i^\circ)) \tag{1.2}$$

From (1.1) it follows that $\alpha_i(t, \tau, t_0, z_i, v)$ are functions measurable in τ and lower semicontinuous in v. Consequently, for any measurable function $v(\tau), \tau \geqslant t_0$, the functions $\alpha_i(t, \tau, t_0, z_i, v(\tau))$ are measurable in τ. From this and from Condition 1, on the strength of the Filippov–Castaing theorem /10/, follows the solvability of equation system (1.2) in the class of measurable functions $u_i(\tau)$ and $m_i(\tau), \tau \geqslant t_0$, taking values from sets $U_i(\tau)$ and $M_i(T)$. If for some $t_* \in [t_0, T]$ we have $h(T, t_*, t_0, z^\circ, v(\cdot)) = 0$, then in (1.2) we set $\alpha_i(T, \tau, t_0, z_i^\circ, v(\tau)) = 0$ for $\tau \in [t_*, T]$ and we choose the controls $u_i(\tau)$ from the Eqs.(1.2) thus obtained.

From the Filippov–Castaing theorem /10/ and Condition 1 follows the possibility of choosing functions $u_i(\tau)$ measurable on the interval $[t_*, T]$. If $\zeta_i(T, t_0, z_i^\circ) \in M_i(T)$, then we set $m_i(\tau) = \zeta_i(T, t_0, z_i^\circ)$ and we choose the controls $u_i(\tau)$ from the equalities (1.2) obtained. The representation

$$\pi_i z(t) = \pi_i\Omega_i(t, t_0) z^\circ + \int_{t_0}^{t} \pi_i\Omega_i(t, \tau)\, g_i(\tau, u_i(\tau), v(\tau))\, d\tau,\quad t \geqslant \tau \geqslant t_0 \tag{1.3}$$

follows from the Cauchy formula. Adding and subtracting the quantity

$$\int_{t_0}^{T} \varphi_i(T, \tau)\, d\tau$$

from expression (1.3) with $t = T$ and allowing for the law for choosing the controls (1.2) when $\zeta_i(T, t_0, z_i^\circ) \overline{\in} M_i(T)$, we obtain

$$\pi_i z(T) = \zeta_i(T, t_0, z_i^\circ)\Big[1 - \int_{t_0}^{T} \alpha_i(T, \tau, t_0, z_i^\circ, v(\tau))\, d\tau\Big] + \int_{t_0}^{T} \alpha_i(T, \tau, t_0, z_i^\circ, v(\tau))\, m_i(\tau)\, d\tau \tag{1.4}$$

However, since $h(T, T, t_0, z^\circ, v(\cdot)) = 0$, a number $i = j$ exists such that the difference within the brackets in (1.4) vanishes. Then

$$\pi_j z(T) = m_j \in M_j(T)$$

When $\zeta_i(T, t_0, z_i^\circ) \in M_i(T)$ this same fact follows from /11/. Theorem 1 has been proved.

Corollary 1. Let $g_i(\tau, u_i, v) = B_i(\tau) u_i - D_i(\tau) v$, where $B_i(\tau), D_i(\tau)$ are matrices of appropriate dimensions, and let the matrices $\pi_i\Omega_i(T, \tau) B_i(\tau)$ be nondegenerated for all $\tau \in [t_0, T]$. Then the pursuers' controls are

$$u_i(\tau) = [\pi_i\Omega_i(T, \tau) B_i(\tau)]^{-1}[\pi_i\Omega_i(T, \tau) D_i(\tau) v(\tau) + \varphi_i(T, \tau) + \alpha_i(T, \tau, t_0, z_i^\circ, v(\tau))(m_i(\tau) - \zeta_i(T, t_0, z_i^\circ))]$$
$$m_i(\tau) \in M_i(T)$$

Here $\alpha_i(T, \tau, t_0, z_i^\circ, v(\tau)) = 0$ for $\tau \in [t_*, T]$, where $h(T, t_*, t_0, z^\circ, v(\cdot)) = 0$. As we see from the analytic notation for $u_i(\tau)$ it is important to find the functions $\alpha_i(T, \tau, t_0, z_i^\circ, v(\tau))$ in explicit form.

Lemma 1. Let the mappings $g_i(\tau, U_i(\tau), v)$ be convex-valued for $\tau \geqslant t_0$, $v \in V(\tau)$, $\varphi_i(t, \tau) \in \Phi_i(t, \tau)$, $t \geqslant \tau \geqslant t_0$. Then

$$\alpha_i(t, \tau, t_0, z_i, v) = \inf_{p \in L_i,\ \varkappa_i(t, t_0, z_i, p) = 1} \{C_i(t, \tau, v, p) + \tag{1.5}$$

$$(p, \varphi_i(t,\tau))\}, \quad t \geqslant \tau \geqslant t_0, \quad v \in V(\tau)$$
$$C_i(t,\tau,v,p) = \max_{u_i \in U_i(\tau)} (-\Omega_i^*(t,\tau)p, g(\tau,u_i,v))$$
$$\varkappa_i(t, t_0, z_i, p) = -C_{M_i(t)}(p) + (p, \zeta_i(t, t_0, z_i))$$
$$C_{M_i(t)}(p) = \max_{m_i \in M_i(t)} (p, m_i)$$

Proof. Condition 1 is equivalent to the following inclusion:

$$0 \in \{\pi_i\Omega_i(t,\tau) g_i(\tau, U_i(\tau), v) - \varphi_i(t,\tau)\} \ \forall v \in V(\tau), \ t \geqslant \tau \geqslant t_0$$

In terms of support functions it is equivalent to the inequality

$$C_i(t,\tau,v,p) + (p, \varphi_i(t,\tau)) \geqslant 0, \quad \forall p \in L_i \tag{1.6}$$

The nonemptiness of the intersection in (1.1) is equivalent to the inequality (Theorem 1 of /3/)

$$C_i(t,\tau,v,p) + (p,\varphi_i(t,\tau)) \geqslant \alpha_i\varkappa_i(t,t_0,z_i,p), \ \forall p \in L_i$$

By virtue of (1.6), when $\varkappa_i(t,t_0,z_i,p) \leqslant 0$ the latter inequality is fulfilled for any nonnegative α_i. If, however, $\varkappa_i(t,t_0,z_i,p) > 0$, then, having set $\varkappa_i(t,t_0,z_i,p) = 1$, we obtain $C_i(t,\tau,v,p) + (p,\varphi_i(t,\tau)) \geqslant \alpha_i$ for all $p \in L_i$, such that $\varkappa_i(t,t_0,z_i,p) = 1$. Hence follows formula (1.5).

2. Let us consider certain sections $m_i(t)$ of the many-valued mappings $M_i(t)$, $t \geqslant t_0$, and let us fix them. We denote

$$\eta_i(t,t_0,z_i) = \pi_i\Omega_i(t,t_0)z_i + \int_{t_0}^{t} \varphi_i(t,\tau)\,d\tau - m_i(t)$$

Let

$$\beta_i(t,\tau,t_0,z_i,v) = \begin{cases} \max\{\beta_i \geqslant 0 : \{\pi_i\Omega_i(t,\tau)g_i(\tau,U_i(\tau),v) - \varphi_i(t,\tau)\} \cap \\ \quad \{-\beta_i\eta_i(t,t_0,z_i)\} \neq \varnothing\}, & \eta_i(t,t_0,z_i) \neq 0 \\ (t-t_0)^{-1}, & \eta_i(t,t_0,z_i) = 0 \end{cases}$$

We introduce the function

$$\mu(t,t_0,z) = \min_{v(\cdot)} \max_i \int_{t_0}^{t} \beta_i(t,\tau,t_0,z_i,v(\tau))\,d\tau$$

and let $\Theta(t_0, z) = \inf\{t: \mu(t,t_0,z) = 1\}$.

Theorem 2. Suppose Condition 1 has been fulfilled and there exist τ-measurable functions $\varphi_i(t,\tau) \in \Phi_i(t,\tau)$, $t \geqslant \tau \geqslant t_0$, and measurable sections $m_i(t)$ of the many-valued mappings $M_i(t)$, $t \geqslant t_0$, such that $\Theta = \Theta(t_0, z^\circ) < +\infty$, and, further, let the greatest lower bound be achieved. Then differential game (0.1) can be completed in time $\Theta - t_0$ from a prescribed initial position (t_0, z°).

Proof. Let $v(\tau) \in V(\tau)$, $\tau \in [t_0, \Theta]$, be an arbitrary measurable function. We denote

$$k(\Theta,t,t_0,z^\circ,v(\cdot)) = 1 - \max_i \int_{t_0}^{t} \beta_i(\Theta,\tau,t_0,z_i^\circ,v(\tau))\,d\tau$$

For τ, $t \geqslant \tau \geqslant t_0$, such that $k(\Theta,t,t_0,z^\circ,v(\cdot)) > 0$ we select the controls $u_i(\tau) \in U_i(\tau)$ from the equations

$$\pi_i\Omega_i(\Theta,\tau)g_i(\tau, u_i(\tau), v(\tau)) - \varphi_i(\Theta,\tau) = -\beta_i(t,\tau,t_0,z_i^\circ,v(\tau))\eta_i(\Theta,t_0,z_i^\circ)$$

if $\eta_i(\Theta,t_0,z_i^\circ) \neq 0$ or, otherwise, from the equations

$$\pi_i\Omega_i(\Theta,\tau)g_i(\tau,u_i(\tau),v(\tau)) - \varphi_i(\Theta,\tau) = 0$$

Condition 1 and the Filippov - Castaing theorem /10/ ensure the possibility of such a selection in the class of measurable functions. Arguments analogous to the proof of Theorem 1 permit us to conclude that for some i

$$\pi_i z(\Theta) = m_i(\Theta) \in M_i(\Theta)$$

Corollary 2. Let $g_i(\tau, u_i, v) = B_i(\tau)u_i - D_i(\tau)v$ and let the matrices $\pi_i\Omega_i(\Theta,\tau)B_i(\tau)$ be nondegenerated for all $\tau \in [t_0, \Theta]$. Then

$$u_i(\tau) = [\pi_i\Omega_i(\Theta,\tau)B_i(\tau)]^{-1}[\pi_i\Omega_i(\Theta,\tau)D_i(\tau)v(\tau) + \\ \varphi_i(\Theta,\tau) - \beta_i(\Theta,\tau,t_0,z_i^\circ,v(\tau))\eta_i(\Theta,t_0,z_i^\circ)], \ \tau \in [t_0, \Theta]$$

Here $\beta_i(\Theta, \tau, t_0, z_i^\circ, v(\tau)) = 0$ for $\tau \in [t_*, \Theta]$, where $k(\Theta, t_*, t_0, z^\circ, v(\cdot)) = 0$.

Lemma 2. Let the mappings $g_i(\tau, U_i(\tau), v)$ be convex-valued for $\tau \geqslant t_0$, $v \in V(\tau)$; $\varphi_i(t, \tau) \in \Phi_i(t, \tau)$, $t \geqslant \tau \geqslant t_0$, be some τ-measurable sections of the many-valued mappings $\Phi_i(t, \tau)$, and $m_i(\tau) \in M_i(T)$ be some measurable sections of the many-valued mappings $M_i(t)$. Then

$$\beta_i(t, \tau, t_0, z_i, v) = \inf_{p \in L_i,\ (p, \eta_i(t, t_0, z_i)) = 1} \{C_i(t, \tau, v, p) + (p, \varphi_i(t, \tau))\}$$

The proof is analogous to that of Lemma 1.

3. Let $\omega_i(t, \tau)$, $t \geqslant \tau \geqslant t_0$ be some τ-measurable numerical functions. We form the following mappings:

$$F_i(t, \tau, u_i, v) = \pi_i \Omega_i(t, \tau) g_i(\tau, u_i, v) - \omega_i(t, \tau) M_i(t), \quad u_i \in U_i(\tau),\ v \in V(\tau),\ t \geqslant \tau \geqslant t_0$$

$$F_i(t, \tau) = \bigcap_{v \in V(\tau)} F_i(t, \tau, U_i(\tau), v)$$

Condition 2. The sets $F_i(t, \tau)$ are nonempty for all $t \geqslant \tau \geqslant t_0$. If Condition 2 is fulfilled, then the many-valued mappings $F_i(t, \tau)$ are measurable in τ and τ-measurable functions $f_i(t, \tau) \in F_i(t, \tau) \subset L_i$ exist for all $t \geqslant \tau \geqslant t_0$. We fix them and we set

$$\xi_i(t, t_0, z_i) = \pi_i \Omega_i(t, t_0) z_i + \int_{t_0}^{t} f_i(t, \tau)\, d\tau$$

(each $\xi_i(t, t_0, z_i) \in L_i$). We consider the functions

$$\gamma_i(t, \tau, t_0, z_i, v) = \begin{cases} \max\{\gamma_i \geqslant 0 : \{F_i(t, \tau, U_i(\tau), v) - f_i(t, \tau)\} \cap \{-\gamma_i \xi_i(t, t_0, z_i)\} \neq \varnothing\}, & \xi_i(t, t_0, z_i) \neq 0 \\ (t - t_0)^{-1}, & \xi_i(t, t_0, z_i) = 0 \end{cases}$$

$$\nu(t, t_0, z) = \min_{v(\cdot)} \max_i \int_{t_0}^{t} \gamma_i(t, \tau, t_0, z_i, v(\tau))\, d\tau$$

Let $T(t_0, z) = \inf\{t : \nu(t, t_0, z) = 1\}$.

Theorem 3. Let the following assumptions be fulfilled:

1°. There exist τ-measurable nonnegative functions $\omega_i, (t, \tau)$, $t \geqslant \tau \geqslant t_0$, exist such that Condition 2 is fulfilled.

2°. There exist τ-measurable functions $f_i(t, \tau) \in F_i(t, \tau)$, $t \geqslant \tau \geqslant t_0$, such that $T = T(t_0, z^\circ) < +\infty$ and the greatest lower bound is achieved.

3°. The equality

$$\int_{t_0}^{T} \omega_i(T, \tau)\, d\tau = 1, \quad \forall i$$

is fulfilled.

Then differential game (0.1) can be completed in time $T - t_0$ from the prescribed initial position (t_0, z°).

Proof. Let $v(\tau) \in V(\tau)$, $\tau \in [t_0, T]$, be an arbitrary measurable function. We consider the function

$$\sigma(T, t, t_0, z^\circ, v(\cdot)) = 1 - \max_i \int_{t_0}^{t} \gamma_i(T, \tau, t_0, z_i^\circ, v(\tau))\, d\tau$$

For $\tau, t \geqslant \tau \geqslant t_0$, such that $\sigma(T, t, t_0, z^\circ, v(\cdot)) > 0$ we select the controls $u_i(\tau) \in U_i(\tau)$ and the functions $m_i(t) \in M_i(t)$ from the equations

$$\pi_i \Omega_i(T, \tau) g_i(\tau, u_i(\tau), v(\tau)) - f_i(T, \tau) - \omega_i(T, \tau) m_i(\tau) = -\gamma_i(T, \tau, t_0, z_i^\circ, v(\tau))\, \xi_i(T, t_0, z_i^\circ)$$

if $\xi_i(T, t_0, z_i^\circ) \neq 0$ or, otherwise, from the equations

$$\pi_i \Omega_i(T, \tau) g_i(\tau, u_i(\tau), v(\tau)) - f_i(T, \tau) - \omega_i(T, \tau) m_i(\tau) = 0$$

Condition 2 and the Filippov–Castaing theorem /10/ ensure the possibility of such a selection in the class of measurable functions. Using the proof plan of Theorem 1, we get that $\pi_i z(T) \in M_i(T)$ for some i, which proves the theorem.

Notes. 1°. We can take $(t-t_0)^{-1}$ as $\omega_i(t,\tau)$. Then condition 2° in the theorem is automatically fulfilled.

2°. If $M_i(t)=\{0\}$, $t\geqslant t_0$, then Theorems 1, 2 and 3 coincide.

Corollary 3. Let $g_i(\tau, u_i, v)=B_i(\tau)u_i-D_i(\tau)v$, where $B_i(\tau)$, $D_i(\tau)$ are matrices of appropriate dimensions and the matrices $\pi_i\Omega_i(T,\tau)B_i(\tau)$ are nondegenerated for all $\tau\in[t_0, T]$. Then

$$u_i(\tau)=[\pi_i\Omega_i(T,\tau)B_i(\tau)]^{-1}[\pi_i\Omega_i(T,\tau)D_i(\tau)v(\tau)+ f_i(T,\tau)+\omega_i(T,\tau)m_i(\tau)-\gamma_i(T,\tau,t_0,z_i^\circ, v(\tau))\xi_i(T,t_0,z_i^\circ)]$$

Here $\gamma_i(T,\tau,t_0,z_i^\circ,v(\tau))=0$ for $\tau\in[t_*, T]$, where $\sigma(T,t_*,t_0,z^\circ,v(\cdot))=0$.

Lemma 3. Let the mappings $g_i(\tau, U_i(\tau), v)$ be convex-valued, $\tau\geqslant t_0$, $v\in V(\tau)$. Then the formula

$$\gamma_i(t,\tau,t_0,z_i,v)=\inf_{p\in L_i(p,\,\xi_i(t,\,t_0,\,z_i))=1}\{C_i(t,\tau,v,p)+(p,f_i(t,\tau))+\omega_i(t,\tau)C_{M_i(t)}(-p)\}$$

$$t\geqslant\tau\geqslant t_0,\quad v\in V(\tau),\quad f_i(t,\tau)\in F_i(t,\tau)$$

occurs.

The proof is by the proof plan of Lemma 1.

Note. For $m=1$ and $\zeta(T,t_0,z^\circ)\in M(T)$ the time $T=T(t_0,z^\circ)$ for ending game (0.1) coincides with the time yielded by the procedure of Pontriagin's first direct method /11/ in the non-stationary case. For $m=1$ and $\eta(\theta,t_0,z^\circ)=0$ the time $\theta=\theta(t_0,z^\circ)$ for ending game (0.1) as well coincides with the time determined by Pontriagin's first direct method /11/.

Example. A conflict-controlled system has the form

$$z_i^{\cdot}=-a_i(t)z_i+u_i-v,\quad z_i(t_0)=z_i^\circ$$

Here $z_i\in E^s$, $s\geqslant 1$, $a_i(t)$, $t\geqslant t_0$, are continuous nonnegative functions and $\|u_i\|\leqslant b_i(t)$, $\|v\|\leqslant c(t)$, $b_i(t)$, $c(t)$ are, for $t>t_0$, continuous numerical functions such that $b_i(t)-c(t)\geqslant 0$, $t\geqslant t_0$, while M_i°: $z_i=0$, $M_i(t)=\{0\}$, $t\geqslant t_0$. The matrizant is

$$\Omega_i(t,t_0)=\exp\left(\int_{t_0}^{t}a_i(\tau)\,d\tau\right)$$

It is seen that condition 1° is automatically fulfilled. As $\varphi_i(t,\tau)$ we take zero. After computations we obtain

$$\alpha_i(\tau,t,t_0,z_i^\circ,v)=[\|z_i^\circ\|\Omega_i(\tau,t_0)]^{-1}\times[(v,z_i^\circ)+((v,z_i^\circ)^2+\|z_i^\circ\|^2(b_i^2(\tau)-\|v\|^2))^{1/2}]$$

The pursuers' controls are

$$u_i(\tau)=v(\tau)-\alpha_i(\tau_1,t_0,z_i^\circ,v(\tau))\Omega_i(\tau,t_0)z_i^\circ$$

The game ending time $T=T(t_0,z^\circ)$ is finite, for example, if

$$\min_{\|v\|\leqslant 1}\max_i(v,z_i^\circ)>0$$

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Translated by N.H.C.